

Level - 3

JEE Advanced++ Pattern/Solutions

141. (30°C)

Let the final temperature be θ_f .

$$\text{Heat gained by block, } Q_b = m_b S_0 \int_0^{\theta_f} (1 + k\theta) d\theta$$

$$\text{Heat lost by water, } Q_w = m_w S_w (45 - \theta_f)$$

$$Q_b = Q_w$$

$$\Rightarrow m_b S_0 \left[\theta_f + \frac{k}{2} \theta_f^2 \right] = m_w S_w (45 - \theta_f)$$

$$\text{Given : } m_b = 100 \text{ g, } S_0 = 4.2 \times 10^3 \text{ J / kg }^\circ\text{C, } k = 0.1^\circ\text{C}^{-1}$$

$$m_w = 500 \text{ g, } S_w = 4.2 \times 10^3 \text{ J / kg }^\circ\text{C}$$

Solving, we get $\theta_f = 30^\circ\text{C}$

142. (i) 240 min (ii) Never, $\theta_{\max} = 120^\circ\text{C}$

Let the specific heat of water = $S_w \text{ J / kg }^\circ\text{C}$

Let the rate of heat supply by the heater = $Q_h \text{ J / min}$

$$\text{So, } (Q_h)(40) = (20)(S_w)(60 - 20) \quad \dots\dots\dots (i)$$

Now, let us assume that water starts boiling 't' minutes after we started adding water at 20°C at rate $r = 0.2 \text{ L / min}$

$$\text{Then, } (Q_h)(t) = S_w [20(100 - 60) + rt(100 - 20)] \quad \dots\dots\dots (ii)$$

$$\text{Dividing equation (ii) by equation (i), and } \frac{t}{40} = \frac{800 + (0.2)(t)(80)}{800}$$

$$\Rightarrow t = 200 \text{ min}$$

So, water starts boiling 240 minutes after beginning of the experiment.

Now, if a liquid with properties same as water except with boiling point 150°C is used, equation (ii) becomes

$$(Q_h)(t) = S_w [20(150 - 60) + rt(150 - 20)] \quad \dots\dots\dots (iii)$$

Dividing equation (iii) by equation (i)

$$\frac{t}{40} = \frac{1800 + 26t}{800}$$

Solving this we get $t = -300$

This answer is unacceptable, So, this means that the liquid mixture never boils.

Let us assume that the maximum temperature reached is θ_m .

$$\text{Then, } (Q_h)(t) = S_w [20(\theta_m - 60) + 0.2t(\theta_m - 20)] \quad \dots\dots\dots (iv)$$

Dividing (iv) by (i) and solving we get

$$t = \frac{20\theta_m - 1200}{24 - 0.2\theta_m}$$

To get a valid answer, t must be positive

$$\text{So, } 20\theta_m - 1200 > 0 \text{ and } 24 - 0.2\theta_m > 0 \Rightarrow \theta_m > 60 \text{ and } \theta_m < 120$$

So, clearly, the maximum temperature reached is 120°C

143. (110 g < m < 143 g)

At first, the ice ball sinks. This means that :

Buoyant force on ice ball < Weight of ice ball

$$\Rightarrow (\text{Volume of ice ball}) \times (\text{Density of water}) < 0.9 + m$$

$$\left(\frac{0.9}{0.9} + \frac{m}{11} \right) \times (1) < 0.9 + m$$

$$\Rightarrow m > 110 \text{ gms}$$

Now, the lead and ice ball gain heat from the surrounding water as they are at temperature -42°C . This causes surrounding water to freeze on the outer surface of the ice ball. This happens until the lead and original ice reach a temperature of 0°C . Let the mass of water that freeze be m_0 .

Heat gained by lead and original ice = Heat lost by freezing water

$$(0.9) + (42)(2.4 \times 10^3) = m_0(336 \times 10^3)$$

$$\Rightarrow m_0 = 0.27 \text{ kg}$$

Now, this ball with extra ice frozen on it floats.

This means :

Buoyant force on ice ball > weight of ice ball.

$$(\text{volume of ice ball}) \times (\text{Density of water}) > 0.9 + m + m_0$$

$$\left(\frac{0.9 + m_0}{0.9} + \frac{m}{11} \right) (1) > 0.9 + m + m_0$$

$$\Rightarrow m < 143 \text{ gms}$$

So, range of m is

$$110 \text{ g} < m < 143 \text{ g}$$

144. (-54°C)

Water shows anomalous expansion; the volume of water increases during freezing. If the entire volume of water would have frozen then its volume would have increased by a factor of $\frac{\rho_w}{\rho_{ice}} = 1.1$ and the level of

water in the calorimeter would have increased by $(h/3)(1.1 - 1) = 2.5 \text{ cm}$. But according to the problem, $\Delta h = 0.5 \text{ cm}$, it implies that a part of water has frozen. Thus the calorimeter has ice water mixture at 0°C .

Heat lost by water = Heat gained by ice

$$m_w C_w (T_w - 0) + \Delta m L_f = C_{ice} m_{ice} (0 - T_{ice}) \quad \dots (1)$$

Where Δm is the mass of the water that freezes. As the volume of water changes by a factor of ρ_w / ρ_{ice} on freezing, we have

$$\Delta h A = \left(\frac{\rho_w}{\rho_{ice}} - 1 \right) \frac{\Delta m}{\rho_w} \quad \dots (2)$$

Where A is the cross-section area of the calorimeter

Substituting expression for Δm from eqn. (2) in eqn. (1) and using $m_w = (h/3)\rho_w A$ and $m_{ice} = (h/3)\rho_{ice} A$, we get

$$C_w A \frac{h}{3} \rho_w T_w + L_f A \Delta h \frac{\rho_{ice} \rho_w}{\rho_w - \rho_{ice}} = C_{ice} \rho_{ice} A T_{ice} \frac{h}{3}$$

$$\text{Hence, } T_{ice} = -\frac{L_f}{C_{ice}} \frac{3\Delta h}{h} \frac{\rho_w}{\rho_w - \rho_{ice}} - \frac{C_w}{C_{ice}} \frac{\rho_w}{\rho_{ice}} T_w$$

On substituting numerical values, we get, $T_{ice} = -54^\circ\text{C}$

145.(5)

Let $AC = a$, $BC = b$ and $AB = \ell$

Then, $\ell^2 = a^2 + b^2$

and also, $a = \ell \cos \theta$, $b = \ell \sin \theta$

After heating, let the lengths become a' , b' and ℓ'

Then, $a' = a(1 + \alpha_x \Delta\theta)$ and $b' = b(1 + \alpha_y \Delta\theta)$

So, $\ell'^2 = a'^2 + b'^2 = a^2(1 + \alpha_x \Delta\theta)^2 + b^2(1 + \alpha_y \Delta\theta)^2$

Using the approximation $(1 + x)^2 \sim 1 + 2x$ for small x .

$$\ell'^2 = a^2 + b^2 + 2a^2\alpha_x\Delta\theta + 2b^2\alpha_y\Delta\theta$$

$$\ell'^2 = (a^2 + b^2) \left[1 + \frac{2a^2\alpha_x + 2b^2\alpha_y}{a^2 + b^2} \Delta\theta \right]$$

Now, putting $a^2 + b^2 = \ell^2$, $a = \ell \cos \theta$ and $b = \ell \sin \theta$,

$$\ell'^2 = \ell^2 \left(1 + (2\alpha_x \cos^2 \theta + 2\alpha_y \sin^2 \theta) \Delta\theta \right)$$

Taking square root and using the approximation

$$(1 + x)^{1/2} \sim 1 + \frac{x}{2} \text{ for small } x.$$

$$\ell' = \ell \left(1 + (\alpha_x \cos^2 \theta + \alpha_y \sin^2 \theta) \Delta\theta \right)$$

We can see from this result that the coefficient of linear expansion along AB is :

$$\alpha_{AB} = \alpha_x \cos^2 \theta + \alpha_y \sin^2 \theta$$

Replacing given values, we get

$$\alpha_{AB} = 5$$

146. (i) $x = 0.5m$ & $x = 0.7m$

(ii) Left end : $0.5\Delta\ell$ towards right

Right end : $1.5\Delta\ell$ towards right

The centre of mass of the rod will not move at all because net external force is zero.

Now, consider the portion between $x = 0.2$ and $x = 0.5$

The portion has been cooled, hence there is contraction.

This contraction, $y_1 = 0.3(\alpha\Delta\theta)$

So, $x = 0.2$ shifts to the right by y_1

The portion between $x = 0$ and $x = 0.2$ is heated, so it expands by amount $y_2 = 0.2(\alpha\Delta\theta)$

The left end ($x = 0$) shifts to the right by amount y_2 relative to the point $x = 0.2$

So, the left end shifts to the right by $y_1 - y_2$ to the right. Also, net decrease in length of the portion

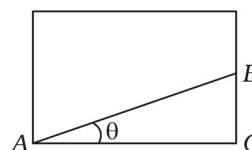
$x = 0$ to $x = 0.5$ is $y_1 - y_2 = 0.1\alpha\Delta\theta$.

Similarly, portion $x = 0.5$ to $x = 0.6$ contracts by $y_3 = 0.1\alpha\Delta\theta$

And portion between $x = 0.6$ and $x = 1.0$ expands by $y_4 = 0.4\alpha\Delta\theta$

So, the right end shifts to the right by amount $y_4 - y_3$.

$$= 0.3\alpha\Delta\theta$$



Now, notice that portion $x = 0.6$ to $x = 0.7$ expands by $0.1\alpha\Delta\theta$.

So, between $x = 0.5$ and $x = 0.7$, there is a contraction by $0.1\alpha\Delta\theta$ and expansion by $0.1\alpha\Delta\theta$.

Hence, the point $x = 0.7$ comes back to its original position.

Total length change $\Delta\ell = -y_1 + y_2 - y_3 + y_4$

$$\Rightarrow \Delta\ell = 0.2\alpha\Delta\theta$$

So, left end shifts to right by $0.1\alpha\Delta\theta$, i.e. $0.5\Delta\ell$ and right end shifts to right by $0.3\alpha\Delta\theta$, i.e. $1.5\Delta\ell$.

The points on the rod that do not shift are $x = 0.5$ and $x = 0.7$.